



# Effects of Additional Interfering Signals on Adaptive Array Performance

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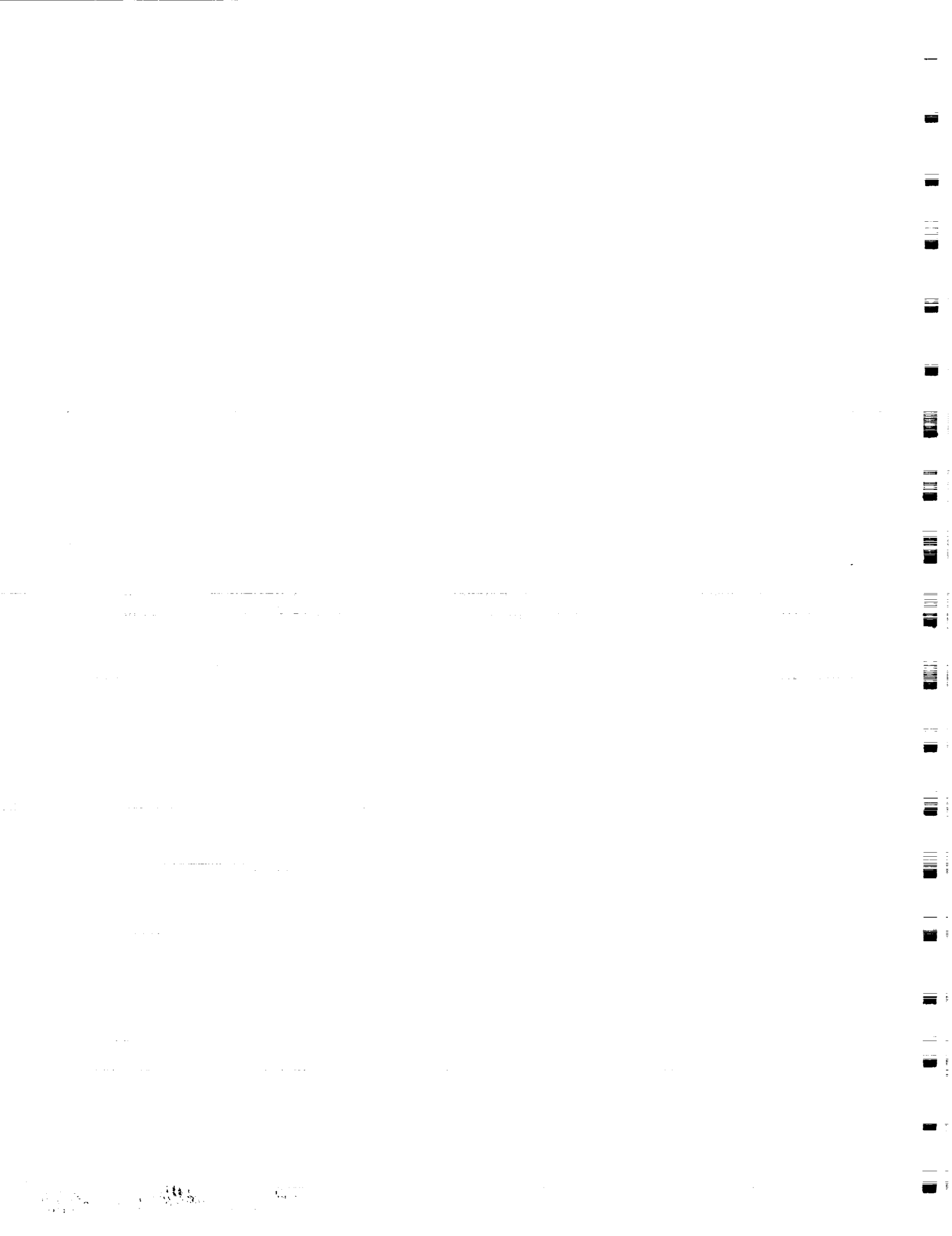
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## 1. Introduction

This report considers the effects of additional interference signals on the performance of a fully adaptive array. Specifically, we consider a steered-beam adaptive array which is used in the case of known direction of arrival for the desired signal. We refer to the case when the number of interference signals exceeds the number of degrees of freedom in the array as the case of "additional interfering signals," where additional means more than the number of degrees of freedom in the array.

This research is motivated by a problem of suppressing weak interference signals by an adaptive array. This problem arises in satellite communications, where interference is caused by transmissions from satellites adjacent to the desired satellite in geostationary orbit [1,2]. These interference signals enter the system through sidelobes in the receive antenna. The interference level is often low; however, because of their similarity to the desired signal these interference signals are coherent to the desired signal, and even small interference signals are objectionable. This interference manifests itself as "ghosts" in a television picture, for example. It is of interest to suppress these weak interference signals by use of an adaptive array.

The suppression of weak interference by an adaptive array has been studied in [1,3,2,4,5]. It has been shown both theoretically and experimentally that effective interference suppression can be achieved by appropriately modifying the weight vector determination algorithm. In this work, we extend those results by considering the effect of additional interference signals on adaptive arrays of this type. While the results presented here are focused on the weak interference suppression problem, they apply to strong interference signals as well.

The performance of adaptive arrays is well-understood when the number of interference signals is less than the number of array elements [6,7,8,9]. However, less is known when the number of interference signals exceeds the number of degrees of freedom in the array. Fujita [10] has studied the array pattern and SNR for a 2-element array with one desired and two interference signals. The behavior of the weights for a similar 2-element example is considered in [9, pp. 84–93].

In this report, we analyze the steady state the performance of the adaptive array in the additional interfering signal case. We first outline the signal environment assumed for this study, and we introduce the steady state weight vectors that are used. We then show analytically that no “simple” change in the weight update algorithm (such as modifying eigenvectors or subtracting known quantities from the covariance matrix) will result in improved array performance as measured by the INR at the array output. This means that performance improvement must be obtained by hardware changes in the array, such as an increase in the directivity of the auxiliary antennas.

We then study the effect of additional interference signals on the performance of the array as a function of antenna directivity. We show that if the auxiliary antennas are sufficiently directive, the array performance in the presence of additional interference signals approach the performance of the system when no additional interference signals are present. The tradeoff between antenna directivity and array performance is analyzed quantitatively.

An outline of this report is as follows. Section II introduces the adaptive array, and outlines the equations which describe the performance of the array. Section III analyzes a simple scenario, and demonstrates that no simple changes to the weight



update algorithm will effectively reduce the performance degradation which results from additional interference signals. Section IV shows quantitatively how directivity of the auxiliary antennas can improve array performance in the additional interference case. Finally, Section V presents the conclusions.

## 2. The Adaptive Array System

We consider an  $N$ -element linear array of equally spaced antennas as shown in Figure 2.1. The array is fully adaptive, so each antenna signal is multiplied by a weight; these weighted signals are then summed to form the array output.

### 2.1 The Signal Environment

Each element in the adaptive array receives a desired signal, a number of interference signals, and noise. The noise present at each element output is zero-mean complex Gaussian white noise with power  $\sigma^2$ . We assume the signals are of sufficiently narrow bandwidth to be well approximated as a single frequency signal. Each signal component is incident on the array at an angle  $\theta_x$  from broadside; this results in an inter-element phase shift  $\phi_x$  of the signal, where

$$\phi_x = 2\pi D \sin(\theta_x)/\lambda, \quad (2.1)$$

$D$  is the inter-element spacing, and  $\lambda$  is the wavelength corresponding to the center frequency of each signal. The subscript  $x$  can either be  $D$  to denote the desired signal, or an  $Ik$ , where  $k = 1, 2, \dots, M$  to denote the  $k$ th interference signal. If

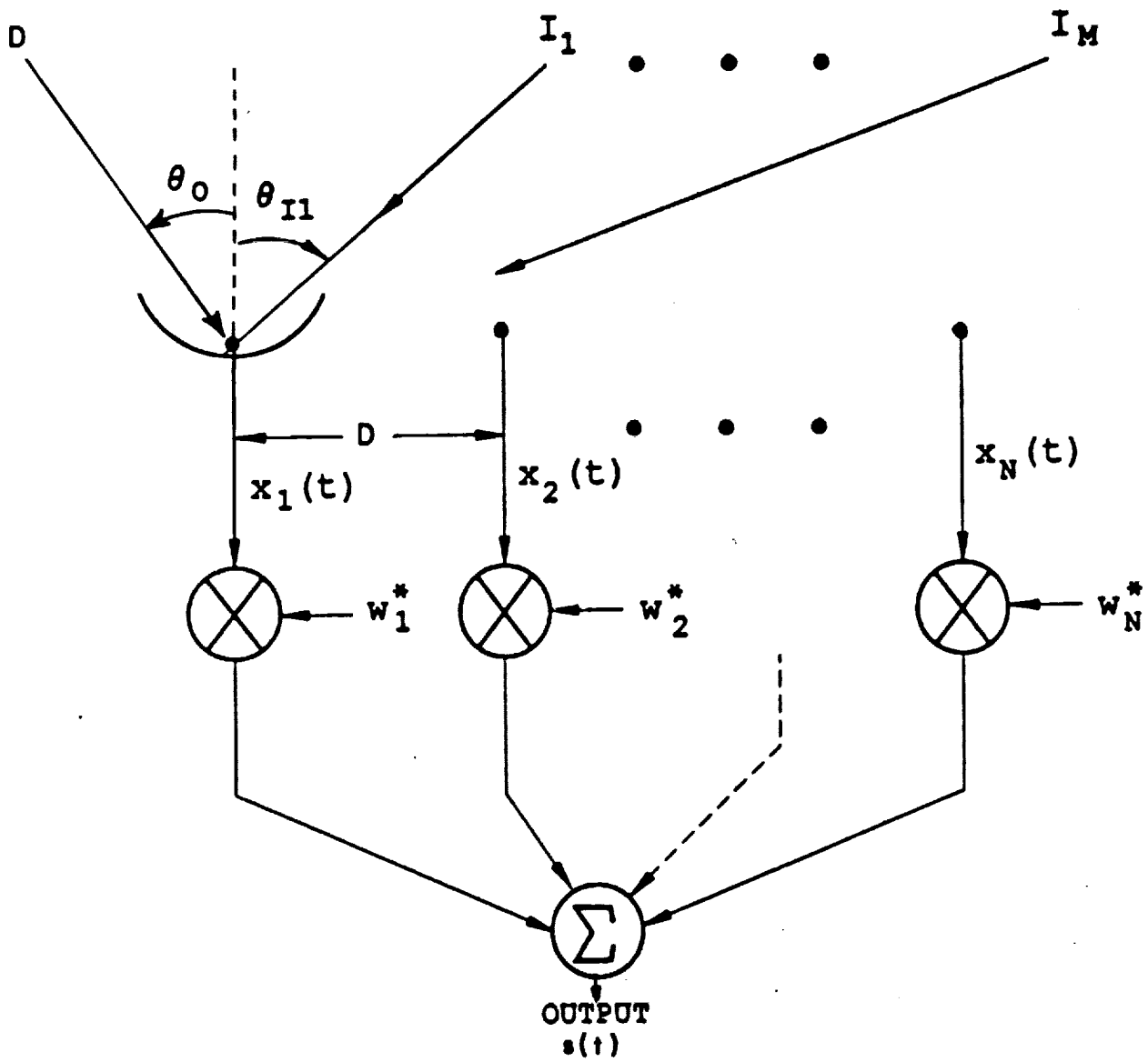


Figure 2.1: The adaptive array configuration.

we assume that the zero phase reference is at the first antenna element, then the element signals are given by

$$x_n(t) = a_{Dn}e^{j[\omega t + (n-1)\phi_D + \psi_D]} + \sum_{k=1}^M a_{Ikn}e^{j[\omega t + (n-1)\phi_{Ik} + \psi_{Ik}]} + \eta_n(t) \quad (2.2)$$

for  $n = 1, 2, \dots, N$ . Here,  $a_{Dn}$  denotes the amplitude of the desired signal at the  $n$ th antenna element, and  $a_{Ikn}$  denotes the amplitude of the  $k$ th interference signal at the  $n$ th antenna element; this amplitude is a combination of the signal amplitude and the antenna gain in that signal direction. The parameter  $\omega$  is the frequency of the signals. The  $\psi_x$  quantities are unknown initial phases associated with the desired and interference signals.

Using vector notation, the array output can be expressed as a vector

$$X(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_N(t) \end{bmatrix} = X_D(t) + \left( \sum_{k=1}^M X_{Ik}(t) \right) + X_\eta(t) \quad (2.3)$$

where

$$X_D(t) = A_D U_D \exp[j(\omega t + \psi_D)], \quad (2.4)$$

$$X_{Ik}(t) = A_{Ik} U_{Ik} \exp[j(\omega t + \psi_{Ik})], \quad (2.5)$$

$$X_\eta(t) = \begin{bmatrix} \eta_1(t) & \eta_2(t) & \cdots & \eta_N(t) \end{bmatrix}^T, \quad (2.6)$$

$$A_x = \begin{bmatrix} a_{x1} & & & 0 \\ & a_{x2} & & \\ & & \ddots & \\ 0 & & & a_{xN} \end{bmatrix} \quad (2.7)$$

$$U_x = \begin{bmatrix} 1 & e^{-j\phi_x} & \dots & e^{-j(N-1)\phi_x} \end{bmatrix}^T \quad (2.8)$$

where  $x = D$  or  $Ik$  for  $k = 1, \dots, M$ , and  $T$  denotes transpose.

The complex weights on the antenna elements are also combined in an  $(N \times 1)$  weight vector

$$W(t) = \begin{bmatrix} w_1(t) & w_2(t) & \dots & w_N(t) \end{bmatrix}^T. \quad (2.9)$$

The received signals  $X(t)$  are weighted and summed to form the array output signal as shown in Figure 2.1;

$$s(t) = W^H(t)X(t) \quad (2.10)$$

where  $H$  denotes complex conjugate (Hermitian) transpose. By inserting equation (2.3) into (2.10), the output signal can be separated into the desired, interference, and noise components:

$$\begin{aligned} s(t) &= W^H(t) \left\{ X_D(t) + \sum_{k=1}^M X_{Ik}(t) + X_\eta(t) \right\} \\ &= s_D(t) + \sum_{k=1}^M s_{Ik}(t) + s_\eta(t) \end{aligned} \quad (2.11)$$

## 2.2 The Steady State Weight Vector

In order to compute array performance measures, one must know the weight vector. In this study we consider the steady-state performance, so we need to know the steady state weight vector. The two steady state weight vectors we will consider is the Wiener (maximum SINR) weight vector, and the modified-Wiener weight vector.

The Wiener weight vector is given by [9]

$$W = \mu \Phi^{-1} S \quad (2.12)$$

where  $\mu$  is a constant,  $S$  is a steering vector given by  $S = A_D U_D$ , and  $\Phi$  is the covariance matrix given by

$$\begin{aligned} \Phi &= E[XX^H] \\ &= E[X_D X_D^H] + \left( \sum_{k=1}^M E[X_{Ik} X_{Ik}^H] \right) + E[X_\eta X_\eta^H] \end{aligned} \quad (2.13)$$

$$= (A_D U_D)(A_D U_D)^H + \left( \sum_{k=1}^M (A_{Ik} U_{Ik})(A_{Ik} U_{Ik})^H \right) + \sigma^2 I \quad (2.14)$$

$$= \Phi_D + \left( \sum_{k=1}^M \Phi_{Ik} \right) + \Phi_\eta \quad (2.15)$$

$$= \Phi_D + \Phi_I + \Phi_\eta. \quad (2.16)$$

It is well-known [7,8,9] that this weight vector maximizes the output SINR of the system, where

$$\text{SINR} = \frac{P_D}{P_I + P_\eta} \quad (2.17)$$

If one is interested in suppressing weak interferences, the Wiener weight vector may not provide sufficiently low INR at the output. In this case, one desires to suppress these interference signals even though they may be weak compared to the noise power. For such applications, a modification of the Wiener weight vector has been proposed [1]. The modified Wiener weight vector is given by

$$W = \mu(\Phi - F\sigma^2 I)^{-1} S \quad (2.18)$$

where  $F$  is a fraction satisfying  $0 \leq F < 1$ . This weight vector maximizes a modified SINR given by [4]:

$$\text{MSINR} = \frac{P_D}{P_I + (1 - F)P_\eta} \quad (2.19)$$

It can be seen from equation (2.19) that the modified SINR is equal to the SINR when  $F = 0$ ; thus, when  $F = 0$  the modified Wiener weight vector reduces to the standard Wiener weight vector. As  $F$  approaches 1, the modified Wiener weights place less and less emphasis on the noise power in the maximization. As a result, more emphasis is placed on minimizing interference power, so weak interference signals are more effectively suppressed.

Because the modified Wiener weight vector is a generalization of the standard Wiener weight vector, we will in the sequel consider only the modified Wiener weight vector. The array performance for the standard Wiener weight vector may be obtained from the results derived below by setting  $F = 0$  there.

### 2.3 Array Performance Measures

From (2.11), the desired, interference, and noise power in the output can be computed. Noting that the desired, interference, and noise components of the signal are mutually uncorrelated, we have

$$P \triangleq E\{|s(t)|^2\} = W^H \Phi W \quad (2.20)$$

where  $\Phi$  is given in equation (2.13). By inserting (2.16) into (2.20), the total power can be partitioned into the sum of the desired, interference, and noise powers:

$$P = P_D + P_I + P_\eta \quad (2.21)$$

$$P_D = W^H \Phi_D W = W^H (A_D U_D) (A_D U_D)^H W \quad (2.22)$$

$$P_I = W^H \Phi_I W = W^H \left( \sum_{k=1}^M (A_{Ik} U_{Ik}) (A_{Ik} U_{Ik})^H \right) W \quad (2.23)$$

$$P_\eta = W^H \Phi_\eta W = \sigma^2 W^H W \quad (2.24)$$

From these powers, array performance measures such as the output interference to noise ratio (INR) and the signal to interference plus noise ratio (SINR) can readily be found.

It is worth noting that to compute the output powers, one needs the signal amplitude and noise power only in the directions of arrival of the desired and interference signals. Equivalently, we may express the output powers in terms of the noise power ( $\sigma^2$ ) and the SNR and INRs of each signal in each antenna element. In computing power ratios, the  $\sigma^2$  terms cancel; thus, power ratios are functions of the SNR and INRs at the elements. We will use this latter formulation in the presentation of the performance results.

### 3. Analysis of an Array with Additional Interfering Signals

From the equations in the previous chapter, we are able to compute the steady-state performance of an adaptive array for any signal environment. In particular, we can compute the performance when the number of interference signals  $M$  is greater than or equal to the number of array elements  $N$ . In this section, we present an analysis of a simple example of this type of scenario.

Consider a 2-element array with the first element as a directional main antenna and the second element as an auxiliary antenna. We assume that there are  $M = 2$  interference signals. Without loss of generality, assume the noise power  $\sigma^2 = 1$ , and for simplicity of notation define:

$$a_D = \text{amplitude of desired signal in the main (= } a_{D1} \text{ in equation (2.7))}$$

$a_{I1}$  = amplitude of interference #1 in the main and auxiliary  
 (=  $a_{I11}$  and =  $a_{I12}$  in equation (2.7))

$a_{I2}$  = amplitude of interference #2 in the auxiliary (=  $a_{I22}$  in equation (2.7))

We assume that the desired signal amplitude in the auxiliary is negligibly small (*i.e.*,  $a_{D2} = 0$  in equation (2.7)), and that the signal amplitude of interference #2 in the main is negligibly small (*i.e.*,  $a_{I21} = 0$ ). With these definitions, we have

$$\Phi_D = \begin{bmatrix} a_D^2 & 0 \\ 0 & 0 \end{bmatrix} \quad (3.1)$$

$$\Phi_{I1} = a_{I1}^2 \begin{bmatrix} 1 & e^{-j\phi_{I1}} \\ e^{j\phi_{I1}} & 1 \end{bmatrix} \quad (3.2)$$

$$\Phi_{I2} = \begin{bmatrix} 0 & 0 \\ 0 & a_{I2}^2 \end{bmatrix} \quad (3.3)$$

$$\Phi_\eta = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (3.4)$$

From equations (2.14) and (2.18), the modified Wiener weight vector is given by:

$$\begin{aligned} W &= \mu(\Phi - FI)^{-1}(A_D U_D) \\ &= \frac{\mu \begin{bmatrix} a_{I1}^2 + a_{I2}^2 + (1 - F) & -a_{I1}e^{-j\phi_{I1}} \\ a_{I1}e^{-j\phi_{I1}} & a_D^2 + a_{I1}^2 + (1 - F) \end{bmatrix} \begin{bmatrix} a_D \\ 0 \end{bmatrix}}{[a_D^2 + a_{I1}^2 + (1 - F)][a_{I1}^2 + a_{I2}^2 + (1 - F)] - a_{I1}^2} \end{aligned}$$



$$= \mu_1 \left\{ a_{I1}^2 \begin{bmatrix} 1 \\ e^{-j\phi_{I1}} \end{bmatrix} + \begin{bmatrix} a_{I2}^2 + (1 - F) \\ 0 \end{bmatrix} \right\} \quad (3.5)$$

where  $\mu_1$  is a constant. Note that the additional interference signal appears as the  $a_{I2}^2$  term in the weight; one can see that if  $a_{I1} \gg a_{I2} + (1 - F)$ , then the effect of the additional interference is small.

From equations (2.22)–(2.24) and (3.5) the output powers can be found:

$$P_D = a_D^2 [a_{I1}^2 + a_{I2}^2 + (1 - F)]^2 \quad (3.6)$$

$$P_I = a_{I1}^2 \left\{ [a_{I2}^2 + (1 - F)]^2 + a_{I1}^2 a_{I2}^2 \right\} \quad (3.7)$$

$$P_n = [a_{I1}^2 + a_{I2}^2 + (1 - F)]^2 + a_{I1}^4 \quad (3.8)$$

Note that  $a_{I2} = 0$  corresponds to the case of only one interference signal; in this case the array is not over-constrained, and  $P_I = (1 - F)^2$ . The output interference power can be made as small as possible by choosing  $F$  close to 1. If  $F = 1$ , perfect interference suppression can be achieved, regardless of how weak this interference signal is.

When the additional interference signal is present (*i.e.*,  $a_{I2} \neq 0$ ), the output INR is given by

$$INR = \frac{P_I}{P_n} = \frac{a_{I1}^2 \{ [a_{I2}^2 + (1 - F)]^2 + a_{I1}^2 a_{I2}^2 \}}{[a_{I1}^2 + a_{I2}^2 + (1 - F)]^2 + a_{I1}^4} \quad (3.9)$$

It can be shown that the INR is maximized for  $F = 1$ . In practice for weak interference signal suppression,  $F$  is chosen near 1, so in the sequel we will set  $F = 1$ . In this case, the INR is given by

$$INR = \frac{a_{I1}^2 a_{I2}^2 [a_{I1}^2 + a_{I2}^2]}{[a_{I1}^2 + a_{I2}^2]^2 + a_{I1}^4}. \quad (3.10)$$

If we further assume that the auxiliary antenna is isotropic so that  $a_{I1} = a_{I2}$ , the output INR becomes

$$INR = \frac{2}{5}a_{I1}^2. \quad (3.11)$$

This value of INR may be excessively high for practical values of  $a_{I1}$ . For example, a typical value of the  $a_{I1}$  corresponds to an INR at the auxiliary of -3 dB [1], which gives from (3.11) an output INR of -7 dB; this is an unacceptably high output INR for some satellite communication applications [1].

Let us consider ways of reducing the performance degradation caused by additional interference signals. One modification is to attempt to remove the effect of interference  $I2$  on the covariance matrix. From equation (3.3) it can be seen that  $a_{I2}$  appears in the lower right element of  $\Phi$ ; thus, if one has an estimate of  $a_{I2}$  obtained from *a priori* knowledge of some sort, one could subtract a fraction of  $a_{I2}$  from  $\Phi$  before forming the weight vector; that is, the weight vector could be determined by

$$W = \mu (\Phi - F\sigma^2 I - G\Phi_{I2})^{-1} S \quad (3.12)$$

where  $G$  is a fraction satisfying  $0 \leq G < 1$ . Note that this modification is similar in spirit to the modification of the standard Wiener weight as presented in (2.18). There is an important difference, however; in the modified Wiener weight method, the noise power  $\sigma^2$  could be estimated from the minimum eigenvalue of the covariance matrix  $\Phi$  under certain conditions [1,4], but no similar method seems to be available for obtaining an estimate of  $\Phi_{I2}$ . Also, it should be pointed out that for  $N > 2$  array elements, the arrival angle of interference  $I2$  must be estimated along with the amplitude. As an example, if  $N = 3$ , then  $\Phi_{I2}$  is of the form

$$\Phi_{I2} = a_{I2}^2 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & e^{-j\phi_{I2}} \\ 0 & e^{j\phi_{I2}} & 1 \end{bmatrix} \quad (3.13)$$

so both  $a_{I2}$  and  $\phi_{I2}$  must be known (or estimated). This information is almost never known *a priori*; if it was known, there would be no need for an adaptive array in the first place. Thus, there seems to be little hope of eliminating the effects of additional interference signals by altering the estimate of the covariance matrix, or by altering the weight determination method.

#### 4. Array Performance Comparisons

Since no practical modification of the weight determination algorithm appears to improve array performance when there are additional interference signals, these interference effects must be reduced by other means. From equation (3.9), it can be seen that the effects of additional interference can be reduced by reducing the signal amplitudes of these interference signals; this can be achieved by using directional auxiliary antennas. This section analyzes the performance of the array as a function of auxiliary antenna directivity for a scenario of practical interest as described in [1].

In this example we consider a 3-element array ( $N = 3$ ) with a desired signal incident at broadside ( $\theta_D = 0^\circ$ ), and two interference signals incident at  $+20^\circ$  and  $-30^\circ$ . The SNR in the main antenna is 14.6 dB, the INR in the main is -5 dB, the

SNR in each auxiliary element is -10 dB, and the INR in each auxiliary element is -3 dB.

The array performance for the case where the number of interference signals is less than the number of array elements has been studied in [1,4]. The basic conclusion of these studies is that when the number of interference signals is less than the number of elements, then for an appropriate value of  $F$ , good interference suppression can be obtained without significantly affecting the output SINR. An example of this case is shown in Figure 4.1, in which the output SINR and INR is shown as a function of the fraction  $F$ . It can be seen that for  $F$  near 1, the output INR can be made very small, while the output SINR stays very close to its maximum value of 14.6 dB (the maximum SINR is attained for  $F = 0$ ). This example is in agreement with the conclusions of the previous section.

When the number of interference signals equals or exceeds the number of elements (*i.e.*, if  $M \geq N$ ), it is not always true that good performance may be obtained. The reason is that an  $N$  element array has  $N$  degrees of freedom; that is, it can satisfy up to  $N$  array gain constraints. For  $M = N - 1$  interference signals, the  $N$  constraints are used to point a beam at the desired signal, and to place  $N - 1$  nulls in the directions of the interference signals. If more interference signals are present, the array does not have any degrees of freedom left place nulls in these additional interference signal directions. As a result, more interference power appears in the output, so the output SINR drops.

Figure 4.2 illustrates the above situation. Here, the desired signal is as in Figure 4.1, and there are four interference signals at  $+20^\circ$ ,  $-30^\circ$ ,  $+50^\circ$ , and  $-60^\circ$ . The SNR in the antenna is 14.6 dB, the INR in the main is -5 dB, the SNR in each

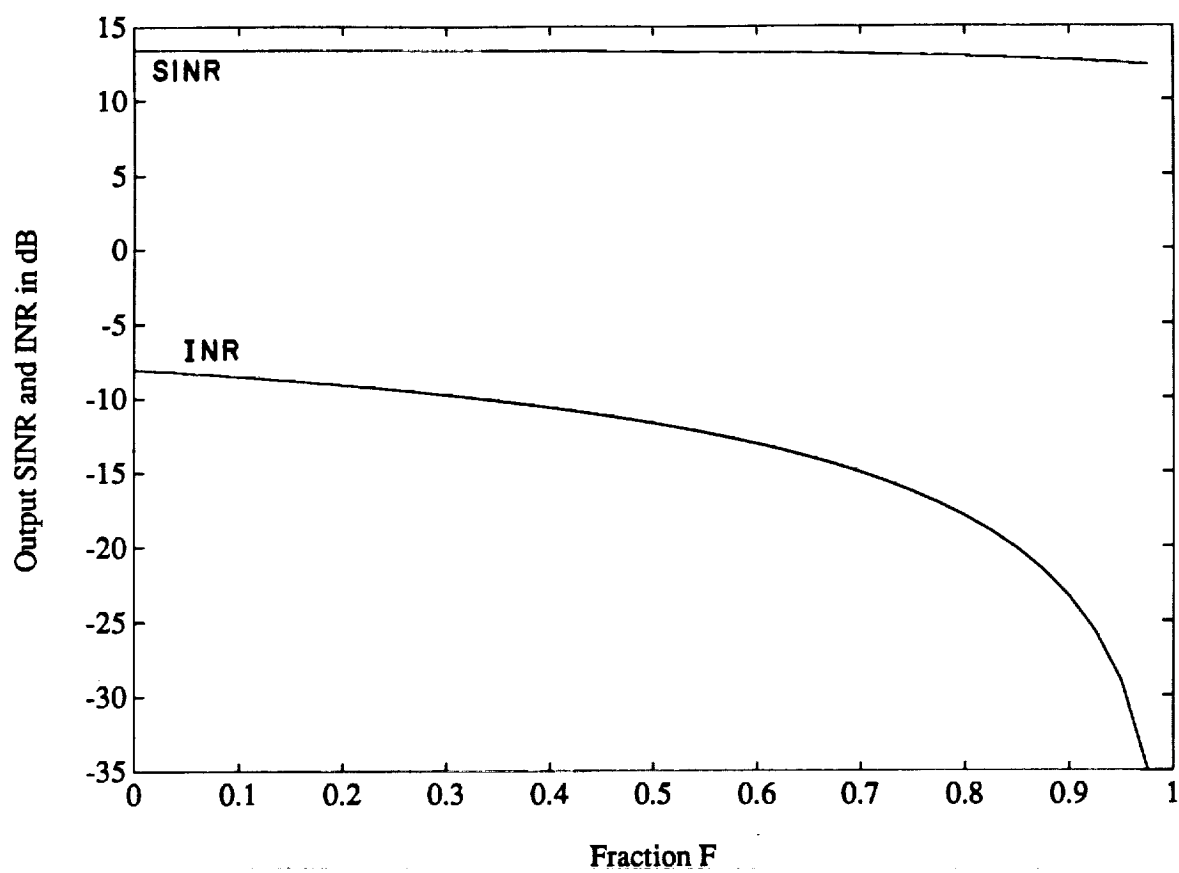


Figure 4.1: Output SINR and INR for a 3-element array with two interference signals

auxiliary element is -10 dB, and the INR in each auxiliary element is -3 dB. This figure shows SINR and INR for arrays with  $N = 3, 4, 5$ , and 6 elements. It is clear that the performance is “good” for  $N = 5$  and 6, but drops significantly for  $N = 3$  and 4.

When the number of interference signals exceeds the number of array elements, improved performance can be achieved by using directive auxiliary antenna elements. Figures 4.3 and 4.4 quantify this statement. In Figure 4.3 the array has three elements, and there are three interference signals (in addition to the desired signal); the desired signal is incident at broadside as before, and the three interference signals are incident at  $+20^\circ$ ,  $-30^\circ$ , and  $+50^\circ$ . We denote the first two interference signals as primary interference signals, and the third as a secondary interference. (For the satellite communication application, one can consider the two primary interference signals as originating from the two satellites adjacent in the geostationary orbit to the desired signal satellite. The additional interference signals originate from satellites in the geostationary orbit, but farther away from the desired signal satellite.) The SNR and INR values for the main and primary interference signals are as in Figure 4.2. The INR of the secondary interference in the main is  $-100$  dB (to model the fact that the highly directive main antenna has very low sidelobes in the region of the secondary interference signals), and the INR of this signal in the auxiliaries varies for the different curves. This variation of secondary INR in the auxiliaries corresponds to differing amounts of directivity of the auxiliary antenna patterns. It can be seen that for highly directive auxiliaries, the performance of the array is essentially equivalent to the 2-interference case; that is, the performance is good, and corresponds to the performance of an array with sufficient degrees of

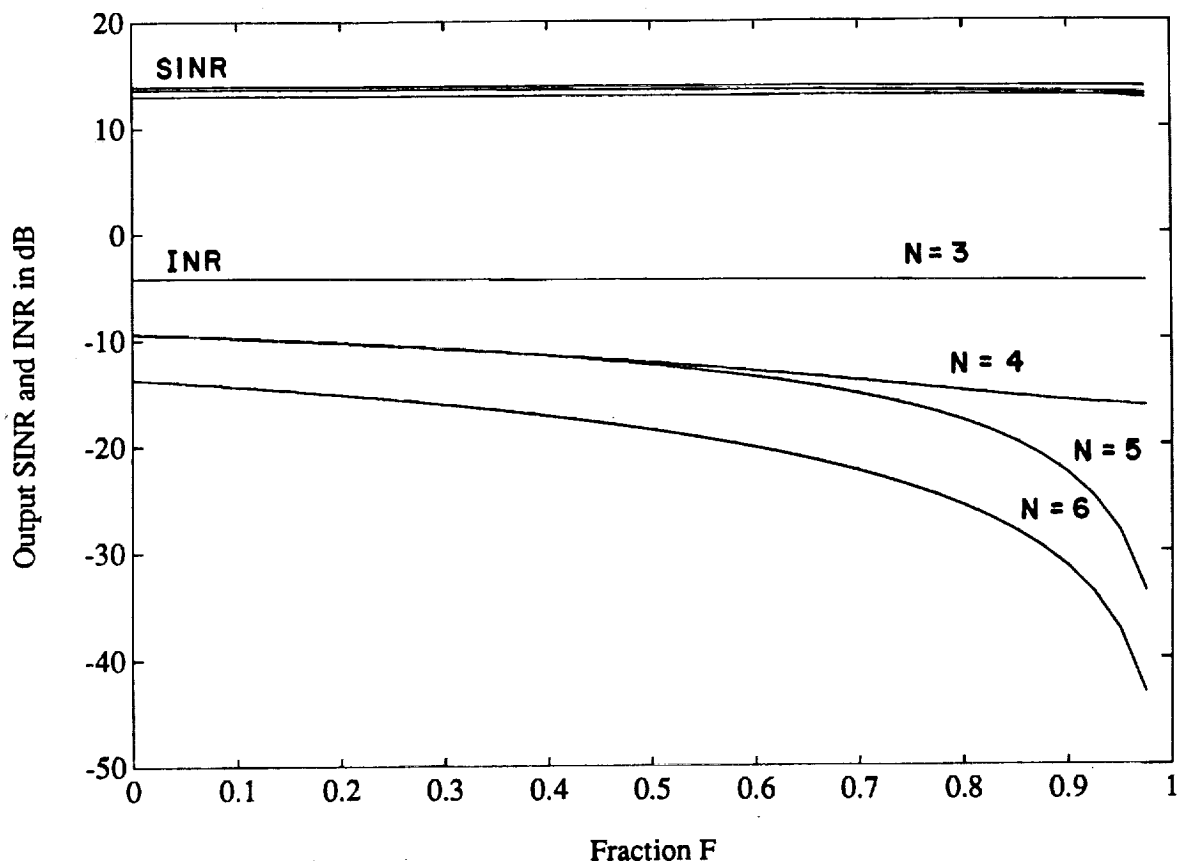


Figure 4.2: Output SINR and INR for a 3-element array with four interference signals

freedom to place nulls in the primary interference directions. As the auxiliaries become less directive, the performance degrades. When the secondary INR is at  $-10$  dB, the auxiliary antennas are isotropic, and the output INR performance is comparable to that in Figure 4.2.

Figure 4.4 is a similar case to Figure 4.3, but using four interference signals, two primary interference signals as before, and two secondary interference signals incident at  $+50^\circ$  and  $-60^\circ$ . By comparing these two figures, it can be seen that the number of secondary interference signals has less effect than the INR level of these signals.

From both Figures 4.3 and 4.4, one sees that a secondary INR of about 10 dB below the primary INR is an approximate threshold for good output INR performance; if the secondary INR is below this value, good array performance is obtained, and if the secondary INR is above this value, the array is over-constrained and cannot effectively suppress all of the interference power.

Finally, Figure 4.5 shows the array performance as a function of the directivity of the auxiliary elements. This figure considers the same scenario as above. In this case we have set  $F = 0.9$  and varied the directivity of the auxiliary elements. A desired signal and two primary interference signals are present as in the previous example, and 1-4 secondary auxiliary signals are incident at  $+50^\circ$ ,  $-60^\circ$ ,  $+70^\circ$ , and  $-75^\circ$ . The INR in the auxiliaries of the secondary interference signals is set to  $-20$  dB. Each auxiliary antenna is considered to be pointing in the general direction of one of the primary interference signals, so in each auxiliary the INR of one primary interference signal is higher than the INR of the other interference signals by the amount shown on the x-axis of Figure 4.5); the INR of the other interference



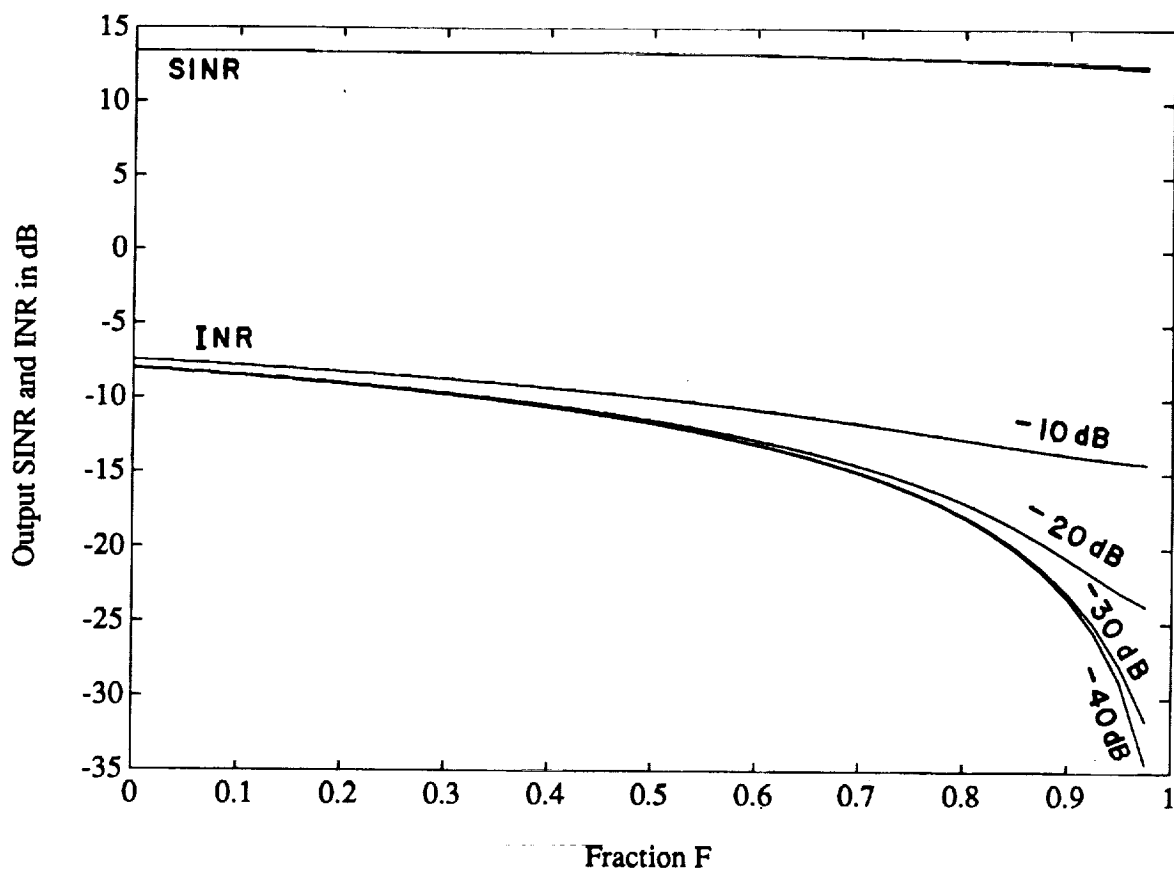


Figure 4.3: Output SINR and INR for a 3-element array with three interference signals, as additional interference INR varies.

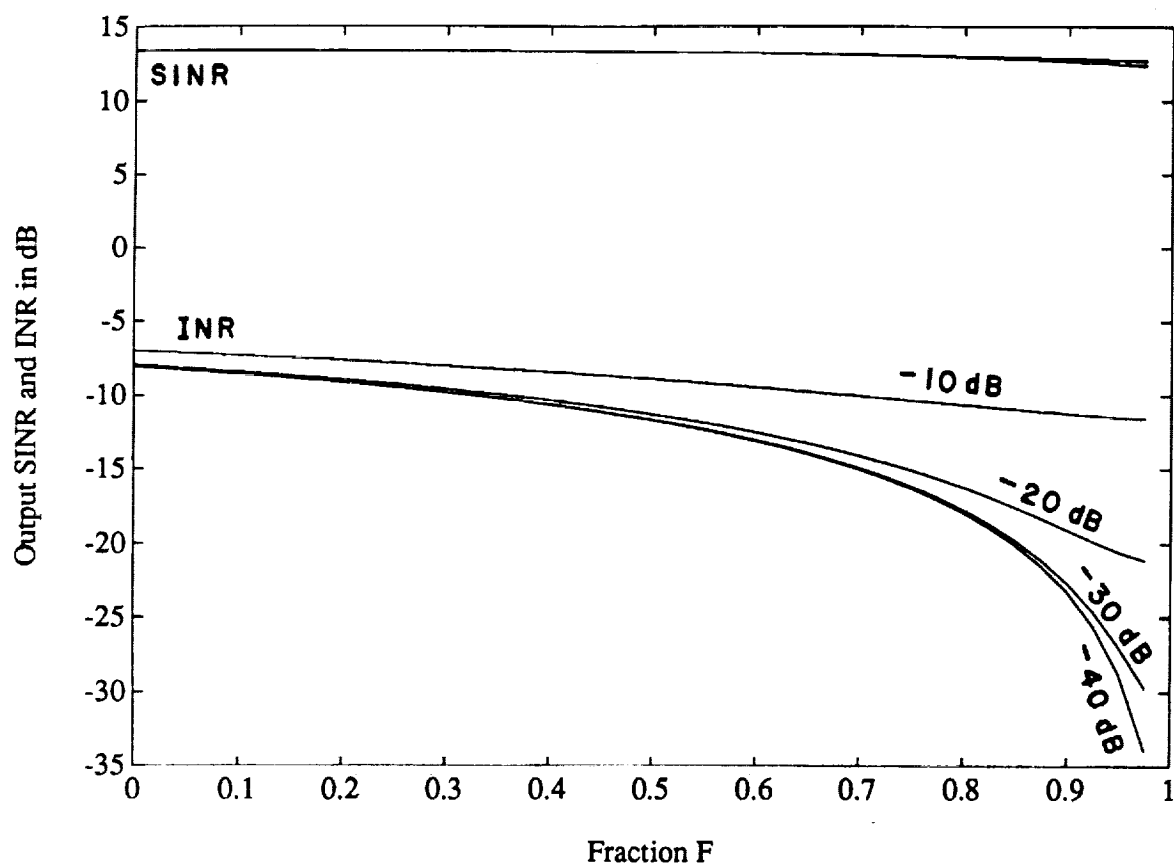


Figure 4.4: Output SINR and INR for a 3-element array with four interference signals, as additional interference INR varies.

signals is fixed at  $-20$  dB. Thus, the  $0$  dB directivity point on Figure 4.5 represents isotropic auxiliaries, and the directivity of the auxiliaries increase as one moves to the right on the figure. It can be seen that when the auxiliary gain is  $20$  dB higher in the primary interference direction, the output INR decreases by about  $10$  dB in all three cases. Thus, antenna directivity is effective at reducing the effects of additional interference signals, even when the interference is weak.

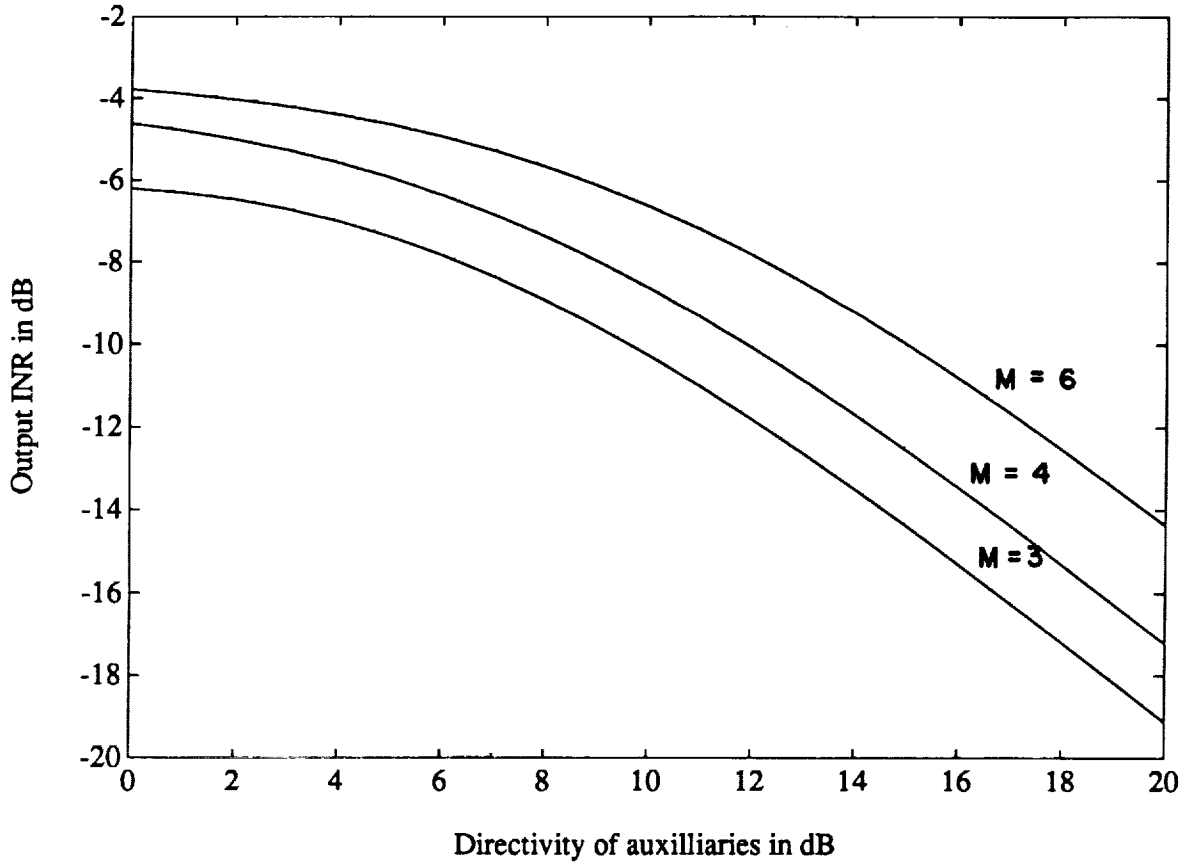


Figure 4.5: Output INR for a 3-element array with  $F = 0.9$  and three, four, and six interference signals, as auxiliary antenna directivity varies.

## 5. Conclusions

We have considered the effect of additional interference signals on the effect of an adaptive array. In particular, we consider an array whose goal is to suppress weak interference signals; this problem is motivated by an application in receiving television signals from geostationary satellites. We studied the steady-state performance of such an adaptive array system when the number of interference signals exceeds the number of array elements, and thus exceeds the number of degrees of freedom available to the array.

It was shown that even when there are more interference signals than array elements, satisfactory suppression of weak interference signals can result if the auxiliary elements are directive. It was shown that if the auxiliaries have about 20 dB gain in the direction of the primary interference signals (the interference signals which have the strongest input power in the main antenna), that the output INR can be reduced by about 10 dB from the isotropic auxiliary level. Thus, in the case of additional interference signals, effective interference suppression results by using directive auxiliaries, even when the goal is suppression of weak interference to well below the noise level.

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